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# Climate Signal and Weather Noise

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# CLIMATE SIGNAL AND WEATHER NOISE

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**ABSTRACT**

A signal of small climate change in either the real atmosphere or a numerical simulation of it tends to be obscured by chaotic weather fluctuations. Time-lagged covariances of such weather processes are used to estimate the sampling errors of time average estimates of climate parameters. Climate sensitivity to changing external influences may also be estimated using the fluctuation dissipation relation of statistical mechanics. Answers to many climate questions could be provided by a realistic stochastic model of weather and climate.

## 1 INTRODUCTION

In any discussion of weather and climate it is, first of all, important to distinguish clearly the meaning of these terms. Climate is usually considered to be in some sense the average weather. Here averages will be considered to be taken over a hypothetical ensemble of earths with specified external influences, but each with its own evolving weather patterns. If the external influences are fixed except for a specified annual cycle, then the resulting climate is expected to be fixed or stationary except for an induced annual cycle. As has been said "Climate is what you expect, weather is what you get." Thus the climate is the hypothetical probability distribution from which weather samples are drawn to produce weather statistics.

There is much current interest in the possibility that the underlying climate probability distribution is changing owing to changes in external influences such as changes in concentrations of greenhouse gases that change the radiative properties of the atmosphere or changes in land surface features that change the thermal properties of the earth's surface. Numerical simulations of the climate system that generate artificial but realistic evolving weather patterns are used to predict the climatic consequences of artificially imposed changes in external influences. Or the real weather records are examined in a search for real climate changes induced by observed real changes in external influences.

Whether examining real or simulated weather records for evidence of a change in the climate there is obviously a sampling problem of distinguishing the signal of climate change from the noise of weather statistics. Enough is known about the time-lagged covariance of weather processes to estimate the magnitude of the weather noise. In the next Section such estimates will be reviewed.

The observed time-lagged covariance of weather processes can also be used to estimate the sensitivity of climate response without the use of numerical simulations but by making use of the fluctuation dissipation relation of statistical mechanics. This relation, as will be discussed in Section 3, is not strictly valid for the climate system and must be treated as an approximation.

The weather sampling problem could, of course, be avoided if one were to construct true climate models that deal directly with the probabilistic properties of the climate system rather than to require sampling from a weather model. Such an approach has been taken with moderate success in the devising of stochastic models of turbulence, a far simpler problem. In Section 4 is described a crude stochastic model of the global atmosphere that captures enough of the observed space and time statistics of the weather to provide encouragement that such an approach may be feasible.

## 2 WEATHER NOISE

Sampling errors associated with a finite time average estimate of any weather variable depend on the time-lagged covariance of the random process given by the time series of observations of that variable. The point is, of course, that successive observations are not independent and their dependence must be taken into account. In application to climate mean statistics this problem has been discussed in detail by Leith (1973) and by Jones (1975). But a rough engineering estimate can be made from the concept of effective time between independent samples defined as

$$S = \int R(t)\, dt \qquad (1)$$

where R(t) is the time-lagged correlation.

For a finite time average based on an averaging interval T, one finds an effective number of independent samples as

$$N = T/S \qquad (2)$$

The sampling error variance of the estimate of the mean is reduced then by a factor N from the variance of the original time series.

Since S is observed to be of the order of a week for weather variables, it is to be expected, for example, that there will be sizable fluctuations in seasonal average temperatures from year to year based solely on sampling fluctuations with N=13 even with no change in the underlying climate probabilities.

This is weather noise, and it tends to obscure the estimation of climate probabilities whether from the real atmosphere or from numerical models of it.

Spatial averaging does not help much since spatial correlation lengths are of the order of a megameter, and this is typically greater than the size of regional climate domains of interest.

## 3 CLIMATE SIGNAL

The key problem in climate system modeling is the detection of a change in the underlying climate probabilities in response to changing influences, especially those induced by human activities, such as $CO_2$ concentration in the atmosphere, or land surface properties.

For the climate system, consider the symbolic evolution equation

$$dx/dt = Q(x) + f(t) \qquad (3)$$

where x is a state vector for the system, Q(x) represents complicated nonlinear internal dynamical processes and f is a vector of external forcing influences. We assume that there exists a stationary base climate for which

$$\langle dx/dt \rangle = d\langle x \rangle/dt = 0 \qquad (4)$$

where $\langle\ \rangle$ indicates the population expectation. Without loss of generality we assume for the base climate that $\langle x \rangle = 0$, so that x represents anomalies from the base climate mean.

The simplest question that can be asked about the climate system is its sensitivity as given by the infinitesimal response of $\langle x \rangle$ to an infinitesimal change in f, i.e., the linear probabilistic response of the mean climate.

We look then for the Greens matrix function G(t) such that

$$\delta\langle x\rangle(t) = \int G(t-s)\delta f(s)ds \qquad (5)$$

Note that an impulsive $\delta f(0) = f\delta(0)$ introduced at time t = 0 will induce a jump in $\langle x\rangle = 0$ to $\langle x\rangle(0+)$ which relaxes back to the base state by the relation

$$\langle x\rangle(t) = G(t)\langle x\rangle(0+) \qquad (6)$$

The state vector x may, in a typical numerical model of the atmosphere, have of order a million components. The corresponding Greens matrix provides the linear response of any one component to a perturbation in any other. The determination of all of the matrix elements of G is therefore an overwhelming task by the usual method of making long integrations with numerical models.

In practice, instead of perturbing each component separately, a collective perturbation is introduced such as of sea surface temperature over a domain or of perturbed heating induced by a change in $CO_2$ concentration in the global atmosphere.

An interesting alternative (Leith, 1975; Bell, 1980) is provided by the fluctuation dissipation relation of statistical physics which states simply that

$$G(\tau) = R(\tau) \text{ for } \tau \geq 0 \qquad (7)$$

where $R(\tau) = C(0)^{-1}C(\tau)$ is the multivariate time-lagged regression matrix for the system, $C(\tau) = \langle x(t)x^*(t+\tau)\rangle$ being the time-lagged covariance matrix.

Although the fluctuation dissipation relation can be proved as a theorem for statistical mechanical systems in thermodynamical equilibrium, the climate system is not one of these, and the relation must be taken as an approximation.

It is not yet clear whether such an approximation, based after all on properties of the real atmosphere, is better or worse than that of devising a numerical model of the atmosphere.

In any case, as a part of the validation of any such model, it seems desirable to check that $R(\tau)$ for the model agrees with $R(\tau)$ as observed in the atmosphere, for otherwise it is unlikely that the important Greens matrix, $G(\tau)$, of the model would agree with that of the real atmosphere.

Remember that the dimension of the regression matrix R is great enough to describe the spatial as well as the temporal statistical properties of the atmospheric climate system.

## 4 STOCHASTIC CLIMATE MODEL

It is tempting to try to devise a stochastic atmospheric climate model of the Langevin type, i.e., with random white forcing and specified damping, that mimics all first and second moments as observed in the real atmosphere. The fluctuation dissipation relation would be built into such a model which would thus provide a crude estimate of climate sensitivity. The feasibility of doing so is suggested by the success of a first crude step in which the atmosphere is treated as a homogeneous, isotropic, two-dimensional turbulent fluid with an eddy mixing of potential vorticity.

Define the potential vorticity as

$$q = \Delta\psi - \lambda^2\psi \qquad (8)$$

where $\Delta$ is the Laplacian operator, $\psi$ is the stream function and $\lambda$ is a deformation wavenumber. Eddy diffusion dynamics for the model is given by

$$\partial q/\partial t = D\Delta q - \alpha q + w \qquad (9)$$

where D is an eddy diffusion coefficient, $\alpha$ an eddy damping rate and w is space- and time-white noise forcing. For a particular wavenumber, k, Eqns. (8) and (9) may be written as

$$q_k = -(k^2 + \lambda^2)\psi_k \qquad (10)$$

$$\partial q_k/\partial t = -(Dk^2 + \alpha)q_k + w_k \qquad (11)$$

The stochastic differential equation (11) of Langevin type generates stationary statistics with variance $Q_k$ of $q_k$ given by

$$Q_k = A/(2(\alpha + Dk^2))$$
$$= (A/2D)/(k^2 + \mu^2) \qquad (12)$$

In order to maintain parsimony of parameters it has been found adequate to set

$$\mu^2 = D\alpha = \lambda^2 \qquad (13)$$

The stream function variance, $\Psi_k$, for wavenumber k is given by

$$\Psi_k = (A/2D)/(k^2 + \lambda^2)^3 \qquad (14)$$

where A is a constant, and the two-dimensional velocity variance is given by

$$U(k) = k^2\Psi_k \approx k^2/(k^2 + \lambda^2)^3 \qquad (15)$$

The isotropic energy spectrum has the shape

$$E(k) \approx kU(k) \approx k^3/(k^2 + \lambda^2)^3 \qquad (16)$$

For $x = k/\lambda$, one finds

$$E(k) \approx f(x) = 8(x + x^{-1})^{-3} \qquad (17)$$

which has a maximum at $x = 1$. The transient energy spectrum for the global atmosphere is observed to have a maximum at planetary wavenumber $k = \lambda = 8$, and for such a choice of $\lambda$ Eqn. (16) provides a fair fit.

Consider next the temporal statistics, in particular, the time-lagged height-height correlation. In this model this is proportional to

$$\pi\int k\Psi(k) \ \mathrm{Exp}[-(\alpha + Dk^2)\tau] \ dk$$
$$= \pi\int k\Psi(k) \ \mathrm{Exp}[-\alpha(1+k^2/\lambda^2)\tau] \ dk$$
$$\approx \int(1+k^2/\lambda^2)^{-3} \ \mathrm{Exp}[-\alpha(1+k^2/\lambda^2)\tau] \ kdk$$
$$\approx \int s^{-3} \ \mathrm{Exp}[-\alpha\tau s] \ ds = E_3(\alpha\tau) \qquad (18)$$

With suitable normalization, we find

$$R(\tau) = 2E_3(\alpha\tau) \qquad (19)$$

in terms of the exponential integral, $E_3$. A good fit to the observed height-height correlation is obtained by choosing the parameter $\alpha = 0.187$ /day.

Note that the parameter $\lambda$ is first chosen to fit spatial statistics and then $\alpha$ is chosen to fit temporal statistics. Note also that in this model the amplitude of the variance depends only on the specified strength of the white-noise forcing.

It is clear that such a model is only a crude starting point for the development of models that take into account the observed three-dimensional mean flow and the inhomogeneous nature of the real climate system.

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